Calculation of Atmospheric Loss From Microwave Radiometric Noise Temperature Measurements

C. Stelzried and S. Slobin
Radio Frequency and Microwave Subsystems Section

Microwave propagation loss in the atmosphere can be inferred from microwave radiometric noise temperature measurements. The relevant equations are given and a derivation and calculation is made assuming various physical models. Comparison is made with the commonly used lumped element atmospheric model (isothermal and uniform loss) and the model with linear temperature and exponential loss distributions. The results are useful for estimating the integral inversion differences due to the model selection. This indicates that the commonly used lumped element atmospheric model is a very good approximation with judicious choice of the effective physical temperature. For the worst case comparison, the lumped element model agrees with the variable parameter model within 0.2 dB up to a propagation loss of 3 dB.

I. Summary

Microwave propagation loss in the atmosphere can be inferred from microwave radiometric noise temperature measurements. Conventionally, the total propagation loss ratio L is calculated from measurements of the noise temperature contribution T'' using the relationship $T'' = Tp(1 - 1/L)$ where Tp is a mean effective physical temperature of the atmosphere. This relationship assumes that the propagation loss and physical temperature can be treated as lumped constants. An "incorrect" choice for Tp results in an error in the determination of L. The equation for the radiometric noise temperature contribution due to the propagation path are derived for various combinations of loss and temperature contributions. These are useful to reduce the error in propagation loss determination. Although it may not be practical or necessary to use this technique in most applications, the analysis is valuable for estimating the integral inversion difference due to the model selection. Conversely, the technique can be used to improve the estimate of Tp required for computation of L using the lumped element model.

II. Introduction

Radiometric microwave noise temperature measurements can be used to estimate atmospheric transmission loss (Refs. 1, 2, 3, 4). Treating the atmosphere as a lumped element, the noise temperature contribution is given by

$$T'' = Tp(1 - 1/L) \tag{1}$$

where

L = propagation path loss (absorption only, no scattering), ratio $[L(\text{dB}) = 10 \log_{10} L]$, $\geqslant 1.0$

Tp = atmosphere mean effective physical temperature, Kelvins

The atmospheric loss is usually calculated from radiometric measurements of T'' using an assumed value for TP ($\simeq$260-

280 K). The purpose of this article is to compare values computed for L in this manner with those computed using a more realistic temperature and loss distribution. These results can either be used directly or as a measure of modeling error in the integral inversion.

III. Theory

The translated radiometric noise temperature (Fig. 1) of a source at a temperature T (Ref. 5) propagating through a medium (such as the atmosphere or a transmission line) is given by (assuming $hf \ll kT$, so that the Rayleigh-Jeans law low-frequency approximation to Planck's radiation law is valid and the noise temperature is proportional to noise power)

$$T' = T'' + T/L \tag{2}$$

where

T'' = radiometric noise temperature contribution of the propagation path, K

$$L = e^{\int_0^{\ell} \alpha(x)dx}$$

$\alpha(x)$ = transmission medium propagation constant[1] at x, nepers/m, [=(dB/m)/10 $\log_{10} e$]

and

$$T'' = \int_0^{\ell} \alpha(x)\, T(x)\, e^{-\int_x^{\ell} \alpha(x')dx'} dx$$

$$= \frac{1}{L} \int_0^{\ell} \alpha(x)\, T(x)\, e^{\int_0^{x} \alpha(x')dx'} dx \tag{3}$$

where

$T(x)$ = physical temperature at x, Kelvins

[1] This definition is consistent with radiative transfer theory (Ref. 6) and should not be confused with the transmission line voltage propagation constant (dB/m)/20 $\log_{10} e$.

IV. Models

The first model is the simplest. Assume uniform temperature and propagation constant distributions

$$T(x) = Tp$$

$$\alpha(x) = \alpha$$

Then

$$L = e^{\alpha}$$

and

$$T'' = \frac{\alpha Tp}{L} \int_0^{\ell} e^{\alpha x} dx$$

$$= Tp\,(1 - 1/L) \tag{4}$$

This derivation agrees with Eq. (1).

The second model is the most realistic investigated. Assume for the propagation path linear temperature and exponential propagation constant distributions (Fig. 2)

$$T(x) = T_1 + (T_2 - T_1)\, x/\ell$$

$$\alpha(x) = \alpha_1\, e^{ax} \tag{5}$$

where

T_1, T_2 = physical temperatures of the propagation path at x = 0 and ℓ, Kelvins

α_1, α_2 = propagation constants of the propagation path, at x = 0 and ℓ, nepers/m,

$$a = (1/\ell)\, \ell n\, (\alpha_2/\alpha_1).$$

Then

$$T'' = \frac{\alpha_1}{L} \int_0^{\ell} e^{ax}\, [T_1 + (T_2 - T_1)\, x/\ell]\, e^{\int_0^{x} \alpha_1 e^{ax'} dx'} dx$$

and replacing x with ℓy

$$T'' = \frac{\alpha_1 \ell}{L} \int_0^1 e^{a\ell y} \left[T_1 + (T_2 - T_1)y\right] e^{\frac{\alpha_1}{a}(e^{a\ell y} - 1)} \, dy \quad (6)$$

where

$$L = e^{\frac{\alpha_1 \ell\,[(\alpha_2/\alpha_1) - 1]}{\ell n\,(\alpha_2/\alpha_1) - 1}}, \quad \alpha_1 \ell = \frac{\ell n\, L \ell n\,(\alpha_2/\alpha_1)}{(\alpha_2/\alpha_1) - 1}$$

These models and others are illustrated in Table 1. Computation of T'' is possible in terms of L, T_1, T_2 and the ratio (α_2/α_1) using numerical integration. Conversely, an estimate of the total atmospheric loss L may be calculated using a measured value of T'' and an atmospheric model with parameters T_1, T_2 and the attenuation ratio (α_2/α_1) with iterative solutions. Models 1 and 2 are compared in Fig. 3 assuming $(\alpha_2/\alpha_1) = 10$, $T_1 = 250$ K, $T_2 = 290$ K. For Model 1,

Case 1:

$$Tp = \frac{T_1 + T_2}{2} \quad (7)$$

This provides a rough estimate of the effective physical temperature as the mean of the upper and lower temperatures of the lossy propagation path.

Case 2:

$$Tp = \left(\frac{\alpha_1}{\alpha_1 + \alpha_2}\right) T_1 + \left(\frac{\alpha_2}{\alpha_1 + \alpha_2}\right) T_2 \quad (8)$$

This provides a judicious choice of effective physical temperature, weighted toward the region of higher loss, improving the agreement between models.

Case 3:

$$Tp \simeq A \left(\frac{\alpha_1}{\alpha_1 + \alpha_2}\right) T_1 + B \left(\frac{\alpha_2}{\alpha_1 + \alpha_2}\right) T_2 \quad (9)$$

This formulation can be used for even closer agreement between models. A and B are chosen to minimize the model difference with perturbations in T_1 and T_2. For example, using $(\alpha_2/\alpha_1) = 10$, $L = 10$, and $T_1 = 270$ K (20 K increase) and $T_2 = 290$ K (unchanged) we have from Eq. (6), $T'' = 257.7$ K. Equation (1) is then satisfied with $Tp = 286.3$ K. Similarly for $T_1 = 250$ K (unchanged) and $T_2 = 310$ K (20 K increase), $T'' = 269.0$ K and $Tp = 299.0$ K. Then from Eq. (9), solving two equations with two unknowns,

$$A \simeq 2.0$$
$$B \simeq 0.90$$
$$Tp \simeq 282.7 \text{ K (for } T_1 = 250 \text{ K}, T_2 = 290 \text{ K)}$$

The effect of a different (α_2/α_1) or other changes in T_1 or T_2 requires modification of A and B or further refinement of Eq. (9) as a function of α_1, α_2, T_1, and T_2.

These three cases progressively improve the agreement with the variable parameter model (Model 2) at the expense of increasing complexity. The appropriate model can be selected on the basis of the accuracy required.

These models are all in good agreement at low loss (less than 3 dB). Moderate values of propagation loss can be determined with small error from noise temperature measurements up to about 200 K. At high loss ($L > 10$ dB) the curve of T'' vs L (Fig. 3) flattens out so that very small errors in noise temperature measurement or modeling will result in very large errors in the propagation loss computation.

Now consider the situation where it is desired to determine the total atmospheric loss from radiometric noise temperature measurements T''. For Model 2, Eq. (6) can be used with measured upper and lower temperatures T_1 and T_2 and an assumed attenuation ratio over the region $x = 0$ to ℓ ($\ell \simeq 30$ km for oxygen and $\ell \simeq 10$ km for water vapor). An iterative computer solution can be performed until the value of L is obtained to satisfy Eq. (6). This inversion may not always be practical. An alternative, simpler method uses the lumped element model (Model 1) with a corrected Tp as described before:

$$L = \frac{Tp}{Tp - T''} \quad (10)$$

This is evaluated using the same parameters as used in Fig. 3 and is compared with the variable parameter model (Model 2). The results are shown in Table 2 and Fig. 4 for Tp corrected using cases 1, 2, and 3. For the worst case comparison (Case 1), the lumped element model agrees with the variable parameter model (Model 2) within 0.2 dB up to a loss of 3 dB. Again, Cases 2 and 3 provide better agreement of the expense of added complexity.

V. Conclusion

It is shown that determination of atmospheric loss from microwave radiometric noise temperature measurements is not sensitive to temperature and loss distribution assumptions at low loss ($L < 3$ dB). For the worst case comparison (Case 1), the lumped element model agrees with the variable parameter model (Model 2) within 0.2 dB up to a total loss of 3 dB. With higher losses, accurate inversions can be made by using a model which is closer to reality. Techniques are suggested for improving the atmospheric loss determination from radiometric noise temperature measurements using the lumped element model with corrected Tp or iterative computation of the appropriate integral solution. Although these techniques have not been compared with field measurements, the model comparisons investigated provide an estimate of the atmospheric loss determination error from radiometric measurements.

References

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Table 1. Summary of propagation path noise temperature contribution (T'') calculations

Model \ Parameter	$\alpha(x)$	$T(x)$	L	T''
1	α	T_p	$e^{\alpha\ell}$	$T_p(1 - 1/L)$
2	$\alpha_1 e^{ax}$	$T_1 + (T_2 - T_1)\frac{x}{\ell}$	$e^{\frac{(\alpha_2 - \alpha_1)\ell}{\ln(\alpha_2/\alpha_1)}}$	$\frac{\alpha_1 \ell}{L} \int_0^1 e^{a\ell y + \frac{\alpha_1}{a}(e^{a\ell y} - 1)} [T_1 + (T_2 - T_1) y]\, dy$
3	α	$T_1 + (T_2 - T_1)\frac{x}{\ell}$	$e^{\alpha\ell}$	$T_1(1 - 1/L) + (T_2 - T_1)(1 - 1/\ln L + 1/L \ln L)$
4	$\alpha_1 + (\alpha_2 - \alpha_1)\frac{x}{\ell}$	T_p	$e^{\frac{(\alpha_1 + \alpha_2)\ell}{2}}$	$T_p(1 - 1/L)$
5	$\alpha_2 \frac{x}{\ell}$	$T_1 e^{bx}$	$e^{\frac{\alpha_2 \ell}{2}}$	$T_1 \frac{(T_2/T_1) - 1/L}{[\ln(T_2/T_1)/\ln L] + 1}$
6	$\alpha_1 + (\alpha_2 - \alpha_1)\frac{x}{\ell}$	$T_1 + (T_2 - T_1)\frac{x}{\ell}$	$e^{\frac{(\alpha_1 + \alpha_2)\ell}{2}}$	$\frac{2 \ln L}{L\left(\frac{\alpha_2}{\alpha_1} + 1\right)} \int_0^1 \left[1 + \left(\frac{\alpha_2}{\alpha_1} - 1\right) y\right] e^{\frac{\ln L}{\frac{\alpha_2}{\alpha_1} + 1}\left[2y + \left(\frac{\alpha_2}{\alpha_1} - 1\right) y^2\right]} [T_1 + (T_2 - T_1) y]\, dy$

where

$y = x/\ell$

$\alpha_1, \alpha_2 = \alpha(x)$ at $x = 0, \ell$ $(y = 0, 1)$

$T_1, T_2 = T(x)$ at $x = 0, \ell$ $(y = 0, 1)$

$a\ell = \ln(\alpha_2/\alpha_1)$

$b\ell = \ln(T_2/T_1)$

$$\alpha, \ell = \frac{\ln L \ln(\alpha_2/\alpha_1)}{(\alpha_2/\alpha_1) - 1}$$

Table 2. Comparison of atmospheric loss calculations using variable parameter and lumped element models

T'' (k)	Calculated Atmospheric Loss, L (dB)						
Measured Atmospheric Noise Temperature	[a]Variable Parameter Model (Model 2)	[b]Lumped Element Model (Model 1)			Difference		
		Case 1	Case 2	Case 3	Case 1	Case 2	Case 3
57.1	1.0	1.0	1.0	1.0	0.0	0.0	0.0
139.5	3.0	3.2	2.9	3.0	0.2	0.1	0.0
210.6	6.0	6.7	5.8	5.9	0.7	0.2	0.1
254.4	10.0	12.4	9.5	10.0	2.4	0.5	0.0

[a]T_1 = 250 K, T_2 = 290 K, (α_2/α_1) = 10

[b]Case 1, 2, 3; Tp = 270 K, 286.4 K and 282.7 K respectively.

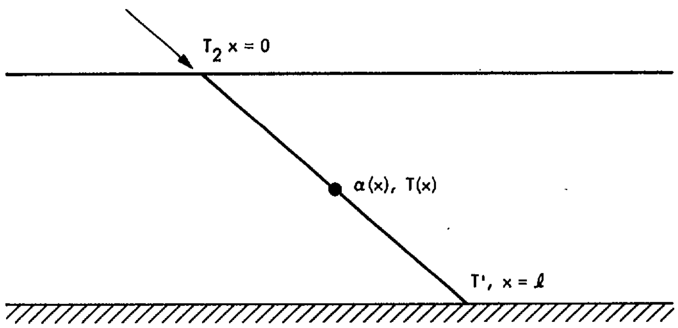

Fig. 1. Thermal noise source, *T*, with propagation constant $\alpha(x)$ and physical temperature *T*(*x*) functions of position *x* along the propagation path, resulting in output noise temperature *T'*

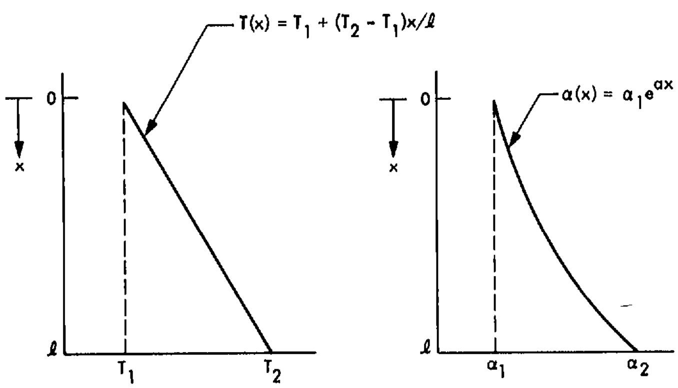

Fig. 2. Representations of temperature and propagation constant distributions in the propagation path for variable parameter model (Model 2)

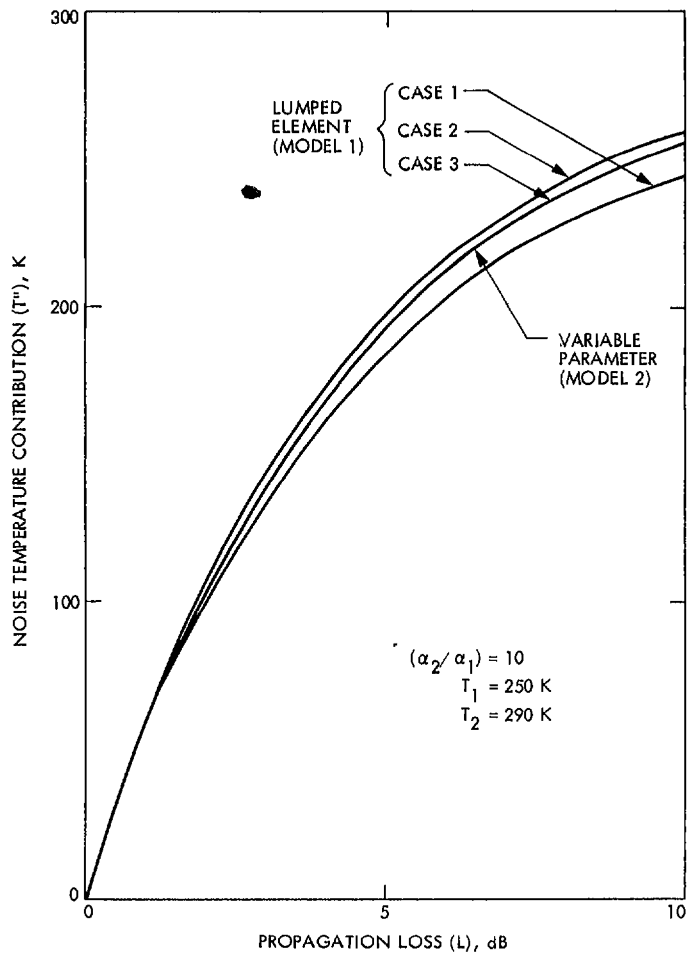

Fig. 3. Noise temperature contribution vs propagation loss

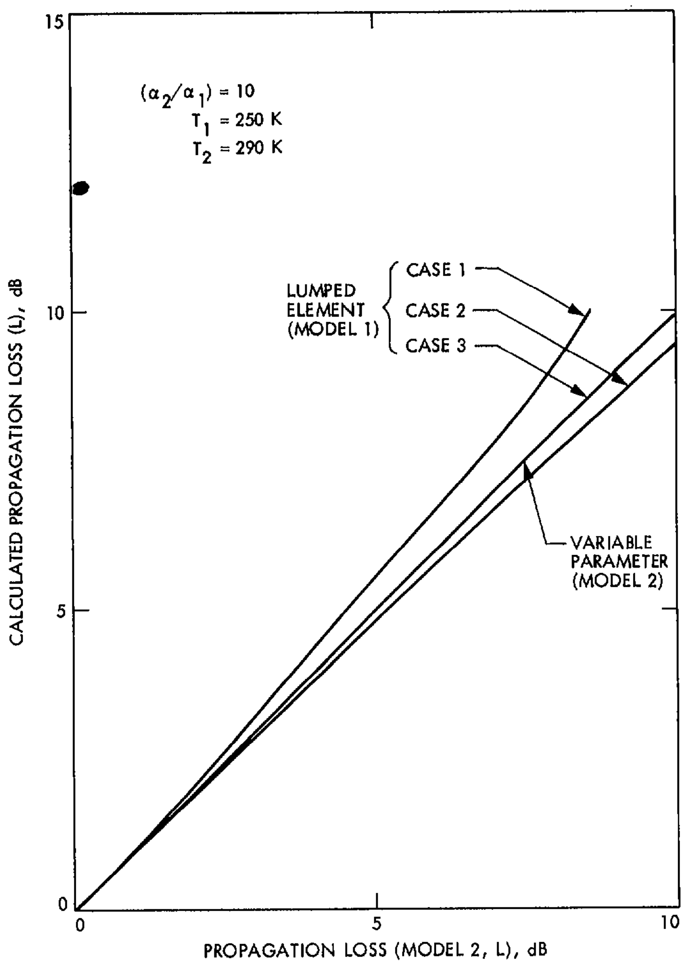

Fig. 4. Propagation loss comparison